CONTROLLING ALLIANCES THROUGH EXECUTING PRESSURE

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ABSTRACT. In this paper the standard prisoners' dilemma is embedded in environmental conditions in which the interaction takes place. This provides a theoretical background to the analysis of the empirical studies which indicate that including additional factors when considering an alliance is very important. We show that such an approach, though theoretically simple, provides a powerful tool for suggesting successful strategies for forming, maintaining and withstanding a rival attack on an alliance, projecting an alliance possible success and determining which measures must be taken to maintain an alliance in a changing environment. It also provides insight into the possibility of preventing alliances in rivals. The relative simplicity of this framework means that the approach can be easily applied when the real life decisions are taken.

1. Introduction

Recently there has been an increasing interest in understanding the instruments responsible for success in business alliances. In a highly competitive and globalized market alliances and joint ventures are becoming increasingly popular by allowing companies to share risks, develop new markets, acquire technological advances and so on. There has also been a significant increase in alliances between competitors^{1,2} as such ventures not only improve their market performance but also reduce costs due to eliminating rivalry hazards. Unfortunately, empirical research²⁻⁸ shows that business alliances have high failure rates (at or higher than 50%). The reason for such failure has a dual nature^{2,9}. On one side non opportunistic reasons such as managerial complexity and coordinating costs play a significant role. The other important reason is interfirm rivalry as each partner is acting to maximize its own benefits. This paper is aimed at addressing the second reason.

In order to explain the cooperative behaviour when the risk of opportunistic behavior is high game-theoretic descriptions of social interactions are often used^{10,11,12}. The single-interaction prisoners' dilemma has been widely used^{11,12} as a generic model of such interactions. Analysis of the single interaction prisoners' dilemma leads to the conclusion that cooperative behaviour cannot be rational; nevertheless it is observed by many empirical studies. The main approach to resolving this paradox has been based on considering the iterated prisoners' dilemma, i.e. considering the game in the long term context^{9,10,11}. This changes the structure of the game and cooperation becomes rational^{13,14}. The basic idea is that by including future interactions "punishing strategies" may be used, i.e., "if you do not cooperate now, you will be punished in the future". Unfortunately,

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the empirical studies indicate¹⁰ that companies using such strategies are not very successful in maintaining alliances. Nor are such strategies evolutionary stable and if a player operates in a noncooperative society no alliance can evolve in the long run¹⁵.

From a game-theoretic point of view there exist many different approaches to overcoming these difficulties 16-24. Such approaches, most of which are based on computer simulations¹⁷⁻²³, in particular include considering complex strategies in an evolutionary model^{17,18}. For example players may be allowed to learn from experience¹⁹. It is shown¹⁹ that mutual cooperation can be maintained when players have a primitive learning ability. It was shown that under proposed learning evolution some cooperative strategies can invade not only unconditional cooperation, Tit for Tat and Pavlov strategies but also noncooperative strategies. Another approach is to embed the prisoners' dilemma into a spatial context^{20,21}. For example²⁰, the version of the iterated prisoners' dilemma with only unconditionally cooperating and unconditionally defecting players interacting with the immediate neighbours was considered. It was shown²⁰ that such a model can generate chaotically changing spatial patterns, in which cooperators and defectors both persist indefinitely. The pattern generated by groups of cooperators exhibits²¹ the scale-invariance which is typical of self-organized criticality²². Multi players games are also considered 23 with solutions found by computer simulations. There is also a direction of research based on the idea of private information available to the players. That is, each player may have some information about themselves or state of the game which is not available to other players. The recent developments 24 in this area have revealed the possibility of cooperation under condition of private information. Providing an important framework for understanding evolution of cooperation in Nature, such approaches however are not easily applicable to the situations when the real life business alliances must be managed. Let us notice nevertheless that all of these studies indicate that cooperative behaviour becomes understandable when the interaction is considered in a wider context (such as length of interaction, for example).

Surprisingly, there are few theoretical studies that suggest modeling cooperation by the games other than prisoners' dilemma. Empirical research on the other side shows that including additional factors when considering an alliance is very important and may suggest successful strategies for forming and maintaining an alliance. For example¹, it is shown that engaging in unilateral commitments significantly increases an alliance success rate. It is argued¹ that unilateral commitments change the payoff structure of the game in such a way that mutual defection is no longer the Nash Equilibrium in the single interaction game. Unfortunately the lack of general game theoretic description of such and similar situations leaves it unexplained why some unilateral commitments lead to an alliance in one situation but not in others.

In this paper we try to overcome this deficiency by embedding the standard Prisoners Dilemma in environmental conditions in which the interaction takes place. We show that such an approach, thought theoretically simple, provides a powerful tool for suggesting successful strategies for forming, maintaining and withstanding a rival attack on an alliance, projecting an alliance's possible success and determining which measures must be taken to maintain an alliance in a changing environment. It also provides insights into the possibility of preventing alliances in rivals. The relative simplicity of this framework means that the approach can be easily applied when the real life decisions are taken.

The main idea of our approach is to evaluate the external conditions under which the alliance is forming. We suggest that the majority of such conditions can be evaluated as either unchangeable or changeable pressure (benefits) to the players. As far as the unchangeable factors are concerned they can be considered as the base payoffs of the single interaction game (for example prisoners' dilemma in our case). We model changeable conditions by introducing a "third player". The "third player" can be irrational²⁵ and, for example, can represent "Nature", that is environmental or other conditions that have an impact on the game but do not depend on the behaviour of the first two players (for example, a highly competitive market or a tax regime imposed by a government). This could also include pressure that a company decides to impose on itself by engaging in unilateral commitments. The "third player" can also be rational. In this context there might be a real third player who interacts with the first and the second players (engaged in a prisoners' dilemma type game for example), possibly to a degree he can choose.

This model correlates well with the empirical studies showing that by considering the two players not in isolation but in the context of their interaction with the "third player" a cooperative Nash Equilibrium between the first two players may emerge. We investigate the effect which the existence of the "third player" has on the possibility of cooperation. This provides mechanisms for controlling and managing the alliances via changing the external conditions. Considering the "third player" as another company attacking the alliance we show how to find an optimal strategy for the attack and also how such an attack could be resisted and the alliance could be saved by good management.

2. Modeling interaction under external pressure

In this section we give an example of modelling an interaction in the context of external conditions. As our example we consider two rival companies engaged in the prisoners' dilemma game who, at the same time, are withstanding an attack from a third player. In the model below we choose certain unchangeable factors such as the value of the market or corporate benefits. Clearly, it is done solely for the purpose of illustrating the approach and the parameters appropriate to the situation under consideration must be taken by the practitioners using this framework. For simplicity we also consider a symmetric base interaction which also can be modified without much difficulty. It is however important to note that the changeable pressure is modelled continuously. Although in the current situation the specific values of such pressure might apply, it is necessary to estimate the impact of the pressure over the whole range of possible outcomes.

We begin by modelling an interaction with the prisoners' dilemma game in which the players simultaneously choose an action from a set of two actions: "Ally" and "Fight". Both players choosing "Ally" results in forming an alliance in which the players share a market (resource) equally and obtain payoff of l. Here 2l represents the value of the market (resource) share available. One player choosing "Fight" and the other "Ally" will be taken as the former player is attacking the latter who does not fight back. Here v represents the value of the market (resource) share transferred as a result of the attack from the "allying" player to the "fighting" player. The parameter x represents extra costs of running the business outside the alliance compared with the cost of running the business in alliance (for example, corporate benefits) If both players chose "Fight" then each player is supposed to have an equal probability of obtaining the whole market (resource). Here f represents the cost

of fighting. In order for this game to be identified as a prisoners' dilemma we must have v > x and v > f and we also assume that $x \ge 0$ and $f \ge 0$. The payoffs for this game are summarized in the table below.

(1)
$$\begin{array}{ccc}
P_1 \setminus_{P_2} & \text{Ally} & \text{Fight} \\
\text{Ally} & {}^l \setminus_{l} & {}^{l-v-x} \setminus_{l+v-x} \\
\text{Fight} & {}^{l+v-x} \setminus_{l-v-x} & {}^{l-x-f} \setminus_{l-x-f}
\end{array}$$

Building on the model let us now assume that there is a third player who attacks the i^{th} player with level q_i , i = 1, 2. For the convenience of the analysis we normalize the levels of the attack to be between zero and one: $0 \le q_i \le 1$. If the third player attacks with level q_i a market share proportional to the appropriate level of attack is lost by the i^{th} player. We assume that there may be a cost of being attacked which is proportional to the level of the attack and will be expressed as $q_i c$, where $c \ge 0$. We also assume that withstanding an attack in an alliance may be easier for a company than on its own, so that there maybe an extra cost $q_i y$, $y \ge 0$, of an attack if companies are not in an alliance. Finally, we suppose that if players are in an alliance they share the cost of the attack equally. These assumptions result in payoffs summarized in the bi-matrix below.

It is fairly simple to obtain the pure strategy Nash Equilibrium conditions, which are summarized in the Table I below.

Table I: Nash Equilibrium conditions for two players.

(3)
$$\frac{\text{NE } P_1 \setminus_{P_2} \quad Ally \setminus_{Ally} \quad Fight \setminus_{Fight} \quad Ally \setminus_{Fight} \quad Fight \setminus_{Ally} }{q_1 \geq \frac{2v - 2x + cq_2}{2v + 2y + c} \quad q_1 \leq 1 - \frac{f}{v} \quad q_1 \geq 1 - \frac{f}{v} \quad q_1 \leq \frac{2v - 2x + cq_2}{2v + 2y + c} }$$
$$q_2 \geq \frac{2v - 2x + q_1 c}{2v + 2y + c} \quad q_2 \leq 1 - \frac{f}{v} \quad q_2 \leq \frac{2v - 2x + q_1 c}{2v + 2y + c} \quad q_2 \geq 1 - \frac{f}{v}$$

This indicates that the cooperative Nash Equilibrium can be obtained if the levels of attack q_i on both players is quite high. If q_i are not high enough the standard non cooperative Nash Equilibrium is observed. If q_i vary significantly the weakly attacked player fights while the strongly attacked player does not fight back. There could also be a mixed strategy Nash Equilibrium as we will show in the example below.

As a next step we consider the rational third player and determine the optimal levels of the attack q_i . In this model we assume that before the game (2) is played the third player chooses q_i or, equivalently, that the first two players are certain about the strength with which they are attacked. This approach simplifies the analysis of the game. The first and the second player make their choices at the game (2) which determine their own payoffs and also the payoff obtained by the third player. The payoffs to the third player will be given in Table II.

Table II: Payoffs to the third player.

Here σ represents the cost of the attack, ε represents the additional cost due to attacking an alliance and ϵ represents the decrement in the cost of the attack if it is made on the companies which are fighting with each other.

We now consider examples showing how the optimal level of attack can be chosen.

3. Example

Consider the game with the following parameters v = 5, x = 2, y = 2, f = 2 and c = 3. Using formulae (3) we obtain the following conditions for the various Nash Equilibria.

Table III: Nash Equilibrium conditions for two players when

(5)
$$\begin{aligned} v &= 5, x = 2, y = 2, f = 2 \text{ and } c = 3. \\ NE^{P_1} \setminus_{P_2} & Ally \setminus_{Ally} & Fight \setminus_{Fight} & Ally \setminus_{Fight} & Fight \setminus_{Ally} \\ q_1 &\geq \frac{3q_2 + 6}{17} & q_1 \leq \frac{3}{5} & q_1 \geq \frac{3}{5} & q_1 \leq \frac{3q_2 + 6}{17} \\ q_2 &\geq \frac{3q_1 + 6}{17} & q_2 \leq \frac{3}{5} & q_2 \leq \frac{3q_1 + 6}{17} & q_2 \geq \frac{3}{5} \end{aligned}$$

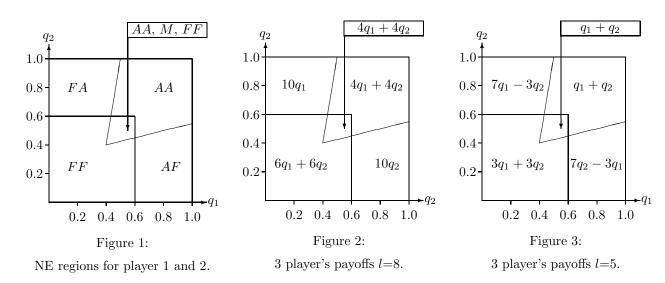
We can also find that if $\frac{3q+6}{17} \leq q_1 \leq \frac{3}{5}$ and $\frac{3p+6}{17} \leq q_2 \leq \frac{3}{5}$ then there is a mixed strategy Nash Equilibrium in which the first player and the second player choose "Ally" with probabilities $\frac{6-10q_2}{7q_2-3q_1}$ and $\frac{6-10q_1}{7q_1-3q_2}$, correspondingly. For each combination of the parameters we have a unique Nash Equilibrium solution except for the region where $\frac{3q+6}{17} \leq q_1 \leq \frac{3}{5}$ and $\frac{3p+6}{17} \leq q_2 \leq \frac{3}{5}$ for which there are three Nash Equilibria: "Ally"-"Ally", "Fight"-"Ally" and the mixed Nash Equilibrium. It can be shown that in this region "Ally"-"Ally" is Pareto efficient meaning that both players obtain their highest payoffs if the "Ally"-"Ally" is played. The Nash Equilibrium regions are shown in Figure 1.

Let us now fix parameters determining the payoff of the third player at $\varepsilon = 1, \epsilon = 1, \sigma = 3$ and consider two scenarios: of high (l = 8) and low (l = 5) value of the market (resource). The payoff to the third player depends on the strategies of the first two players (4). If the Nash Equilibrium is not unique for some range of the parameters q_i we assume that the Pareto efficient Nash Equilibrium is played. The payoffs to the third player and the points at which its maxima are reached are summarized in Table IV below.

Table IV: Payoffs to the third player and points of maxima.

	l = 8			l = 5			
range of $q_1 \& q_2$	NE	payoff	max	at (q_1, q_2)	payoff	max	at (q_1, q_2)
$\frac{3q_2+6}{17} \le q_1$ $\frac{3q_1+6}{17} \le q_2$	AA	$4q_1 + 4q_2$	8	(1,1)	$q_1 + q_2$	2	(1,1)
$q_1 \le \frac{3}{5}$ $q_2 \le \frac{3}{5}$	FF	$6q_1 + 6q_2$	108 17	$\left(\frac{3}{5}, \frac{39}{85}\right)$ $\left(\frac{39}{85}, \frac{3}{5}\right)$	$3q_1 + 3q_2$	$\frac{54}{17}$	$\left(\frac{3}{5}, \frac{39}{85}\right)$ $\left(\frac{39}{85}, \frac{3}{5}\right)$
$\frac{\frac{3}{5}}{5} \le q_1 q_2 \le \frac{3q_1 + 6}{17}$	AF	$10q_2$	$\frac{90}{17}$	$\left(1, \frac{9}{17}\right)$	$7q_2 - 3q_1$	$\frac{24}{17}$	$\left(\frac{3}{5}, \frac{39}{85}\right)$
$q_1 \le \frac{3q_2+6}{17}$ $\frac{3}{5} \le q_2$	FA	$10q_1$	90 17	$\left(\frac{9}{17},1\right)$	$7q_1 - 3q_2$	24 17	$\left(\frac{39}{85},\frac{3}{5}\right)$

We can draw plots (see figure 2 and 3) that represent the payoff obtained by the third player when he chooses different levels of attack. Comparing the values at the points where the maxima are reached, the optimal choice of attack levels is $q_1 = 1$ and $q_2 = 1$ if l = 8. Such a strategy of the third player forces the first two players to form (or maintain) an alliance. If l = 5, we see that the third player obtains maximum payoff at the points $(\frac{3}{5}, \frac{39}{85})$ or $(\frac{39}{85}, \frac{3}{5})$ but if the levels of attack are set to be exactly at these values the payoff bi-matrix (2) for the two firms is non-generic and there exist an infinite number of Nash Equilibria for this game. It includes the "Ally"-"Ally" Nash Equilibrium which, if it is played by the first two players, gives them the highest payoffs. The third player obtains the payoff of $\frac{54}{17}$ only if the first two players choose to fight. Therefore to prevent the first and the second players from swapping between Nash Equilibria the third player must not choose $(\frac{3}{5}, \frac{39}{85})$ or $(\frac{39}{85}, \frac{3}{5})$ but choose $(\frac{3}{5} - \delta_1, \frac{39}{85} - \delta_2)$ or $(\frac{39}{85} - \delta_1, \frac{3}{5} - \delta_2)$ such that these points are still in ("Fight"-"Fight") Nash Equilibrium region and δ_1 and δ_2 are very small. We can see that for this example the optimal choice of parameters q_1 and q_2 does not exist. The best strategy for the third player would be to break the alliance by using appropriate levels of attack. In this case a moderate, asymmetric attack will achieve higher payoff than a strong (symmetric or asymmetric) attack.



It is possible to show that (for v = 5, x = 2, y = 2, f = 2, c = 3, $\varepsilon = 1$, $\epsilon = 1$) if the difference between value of the market l and the price of the attack δ is

- less than -1 (note that in this case the actual cost of the attack is $\sigma \epsilon$ so the difference between value of the market l and the cost of the attack is less than 0) then it is optimal not to attack, that is $q_1 = 0$ and $q_2 = 0$;
- between -1 and $\frac{13}{4}$ then the optimal choice of parameters q_1 and q_2 does not exist and the third player should follow the approach described above;
- greater than $\frac{13}{4}$ then the optimal choice of attack levels is $q_1 = 1$ and $q_2 = 1$; this choice of the third player forces the first and the second players to form (or maintain) an alliance.

The proof can be obtained using item-by-item examination of all possible cases. Similar conditions can be obtained for any generic set of parameters.

4. Summary and discussions.

This simple example leads to a few important conclusions.

Firstly, as it can be seen from the results summarized in figure 1, the successful formation of an alliance greatly depends on the position of the players on the Nash Equilibrium diagram (figure 1). As in a single interaction prisoners' dilemma in the absence of pressure cooperative behaviour is not a Nash Equilibrium and alliances which are formed under such conditions are not stable. However under the threat of an attack by another player (or under unfavorable conditions) the players may be induced to cooperate. It is clear that cooperative behaviour becomes a Nash Equilibrium of the game (2) because an attack on the players changes their payoffs. But on the other hand, if, in modelling, such changes are not taken into account we would obtain the result that cooperation is not rational when, in fact, it is. In the considered model strong pressure, where its level does not vary significantly between the players will stabilize the possibility of forming an alliance. On the other hand, an asymmetric, weak or moderate pressure does not provide an incentive for cooperation.

In this instance the possibility for stabilizing an alliance arises by applying a careful managing strategy. For example, assume the first player position and suppose that $q_1 = 1/5$ (other values of q_1 can be treated in a similar way). Assume also that the first player is interested in forming an alliance. The idea is that the first player may be able to change its own value of q_1 (for example by engaging in unilateral commitments up to some extent) in such a way that the alliance becomes a Nash Equilibrium. Three different approaches must be taken in such a situation depending on the value on q_2 . If $0 \le q_2 \le 3/7$ (3/7 appears as the point of intersection of $q_1 = \left(3q_2 + 6\right)/17$ and $q_2 = \left(3q_1 + 6\right)/17$ both players are in the "Fight"-"Fight" region and the pressure on the second player is not high enough to promote a stable alliance. In this case the first player's best strategy would be "Fight". The unilateral commitments would only shift the players position into "Ally"-"Fight" region and the first company would suffer losses. If $3/7 < q_2 \le 3/5$ both players are still in the "Fight"-"Fight" region but the pressure on the second player is high enough to allow for the possibility of an alliance. As can be seen from the figure 1, the first company may change its own value of q_1 in such a way that "Ally"-"Ally" becomes a Nash Equilibrium. The unilateral commitments in this case can lead to ensuring an alliance, though it should be noted that the level of such commitments must be carefully assessed in order to avoid shifting the players even further to the right on the Nash Equilibrium diagram which would result in "Ally"-"Fight" equilibrium and alliance failure. If $3/5 \le q_2$ the second player is under high pressure and is likely to cooperate. The main threat to the alliance is that the first player might prefer to use such an opportunity to maximize its immediate payoff jeopardizing the future of the alliance. To avoid such a contempt (for example the first company might be interested in a strategic alliance which is expected to be highly beneficial in the future) the first player might engage in unilateral commitments to stabilize the alliance. As we can see engaging in unilateral commitments might be highly beneficial for formation and maintaining of an alliance if used appropriately. Note that it is assumed that the parameters of the model can be estimated accurately enough. Although it is always possible to allow for a margin of error, such an error must not be neglected when a strategy is chosen as it might significantly change the strategy. The analysis of the risks arising from this situation must be done, however we do not address this issue in this paper.

The alliances must be frequently reassessed and managed¹. In this instance the results of the model can also be used for projections of possible outcomes of an alliance. The estimates of future threats and pressures must be made. If a company anticipates the position of an alliance to leave the "Ally"-"Ally" region it should be assessed if it is possible to save the alliance by changing the values of q_1 or (and) q_2 . If it is possible, careful management might save the alliance. If it is not the case the company might save itself from heavy losses in the future by withdrawing from an alliance prior to its failure.

Another result of the above analysis concerns the third player's strategy. As we have seen from the examples, depending on the relationship between the value of the market and the costs of the attack, the third player may either wish to split the alliance or it may be better for the third player to engage in the strongest symmetric attack. A strong attack, where the level of attack does not vary significantly between the target firms, will stabilize an alliance. An asymmetric, weak or moderate attack will result in an alliance failure if the alliance is not managed well. It must be noticed though that in this situation there exist mechanisms implying that good management can always ensure that the alliance withstands an attack. The strategy here is the same as when $3/5 \le q_2$: the weakly attacked player (let us say player one) must increase the value of q_1 to shift the players position to the right on the Nash Equilibrium diagram. This opportunity might be important then the third player represents hostile conditions for one of the partners which are of a temporary nature. In this situation the temporary sacrifice of another player might save the alliance from failure bringing strategic benefits in the future.

As it can be seen from the example we have considered, a simple idea of including environmental conditions into consideration provides several important implications which would be overseen otherwise. In the considered example, in addition to explaining the mechanisms which might be responsible for formation of alliances, one of the important benefits is to allow us to distinguish between two "Fight"-"Fight" situations. In one unilateral commitments are beneficial and in another would imply losses. These allow us to use the proposed approach for determining successful strategies for forming and maintaining alliances.

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